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# Charge-Independence Breaking in the Two-Pion-Exchange Nucleon-Nucleon Force

by

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## Abstract

Charge-independence breaking due to the pion-mass difference in the (chiral) two-pion-exchange nucleon-nucleon force is investigated. A general argument based on symmetries is presented that relates the charge-symmetric part of that force to the proton-proton case. The static potential linear in that mass difference is worked out as an explicit example by means of Feynman diagrams, and this confirms the general argument.

Although a complete understanding of isospin violation (IV) in the nuclear force remains to be achieved, significant progress has been made. Decades of experimental progress (reviewed and summarized nicely in Refs.[1, 2, 3]) have been supplemented recently by the advent of chiral perturbation theory (ChPT)[4, 5, 6, 7]. This powerful technique casts the symmetries of QCD into effective interactions of the traditional, low-energy degrees of freedom of nuclear physics (viz., nucleons and pions). These building blocks (Lagrangians) can then be combined in the usual way to produce IV nuclear forces.

One of the significant attributes of effective field theories is power counting, which is used to organize calculations[4, 5, 6, 7, 8, 9]. That is, a well-defined ordering of terms in the Lagrangian according to an intrinsic-size criterion is used to generate all nuclear-force terms of a particular size. In addition, IV terms in such theories can be classified according to whether their origin is the mass differences of the quarks or hard electromagnetic (EM) interactions at the quark level[5]. Soft EM interactions (such as the Coulomb force) can be constructed in the usual way[10].

This scheme was used recently for the first time[5] to explain the sizes of the different forms of IV in the nuclear force. A convenient and universal[1] classification for nuclear isospin is: class (I) - isospin conserving; class (II) - charge-independence breaking (CIB) of isospin, but charge symmetric; class (III) - charge-symmetry breaking (CSB) of isospin; class (IV) - isospin mixing in the  $np$  system between  $T = 0$  and  $T = 1$ . Power counting can be used to demonstrate[5] that class (I) forces are stronger than class (II), which is stronger than class (III), which is stronger than class (IV). Thus, class (II) isospin violation is the largest, and that is the purview of this work.

Many mechanisms contribute to charge-symmetric CIB, but the largest is due to the mass difference of the charged and neutral pions,  $\delta m_\pi = m_{\pi^\pm} - m_{\pi^0}$ , which is primarily of electromagnetic origin. Indeed most CIB mechanisms are of this type. The pion-mass difference produces an IV effect of order  $(\delta m_\pi/m_\pi) \sim 3\%$  in the one-pion-exchange potential (OPEP). Simultaneous pion-photon exchange[10] is of order  $(\alpha/\pi)$  times OPEP, as would be the effect of EM modification[11, 12] of the  $\pi$ - $N$  coupling constant (not yet detected); both of the latter are class (II) mechanisms. In addition there will be short-range CIB forces, of nominal size  $(\alpha/\pi)$  times the usual short-range force; such forces have been discussed recently[13] in the context of meson-exchange models of CIB. Finally, in the two-pion-exchange potential (TPEP) the different pion masses generate the dominant CIB[14], which is also of nominal size  $(\alpha/\pi)$  times OPEP.

We present below a general argument for the class (II) isospin violation that is produced by differing pion masses in the two-pion-exchange nucleon-nucleon force.

The general argument (which applies to arbitrary radial forms, including the leading order, subleading order, ... in ChPT) will be supplemented by a specific example, namely the static limit (leading order in ChPT) of that potential, in order to validate the general argument. We also note that the general argument can be easily incorporated in partial-wave analyses such as that carried out by the Nijmegen group in their analysis of nucleon-nucleon scattering[15]. Finally, we estimate the effect of the leading-order IV on the  $^1S_0$  scattering lengths.

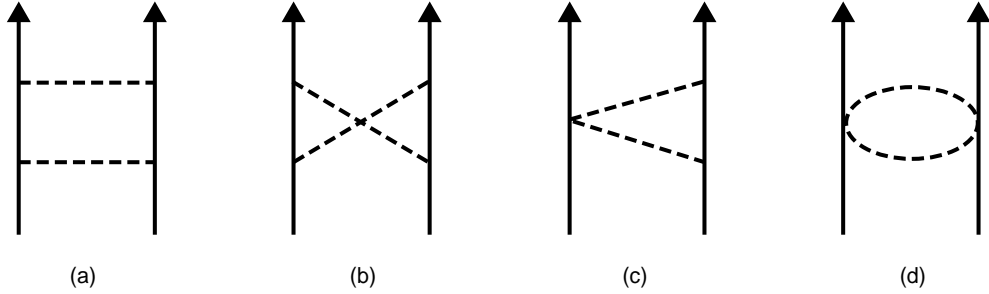


Figure 1: Two-pion-exchange graphs for nucleon-nucleon scattering.

Two-pion-exchange isospin-conserving forces are an old problem with a new twist. In static order (containing only terms that remain when the nucleon mass,  $M$ , or the large-mass scale of QCD,  $\Lambda$ , becomes very large) the diagrams of Fig. (1) contribute to the TPEP. The vertices and propagators follow from the leading-order Lagrangian for pions and nucleons,

$$\mathcal{L}^{(0)} = \frac{1}{2}[\dot{\vec{\pi}}^2 - (\vec{\nabla}\vec{\pi})^2 - m_\pi^2\pi^2] + N^\dagger[i\partial_0 - \frac{1}{4f_\pi^2}\vec{\tau} \cdot (\vec{\pi} \times \dot{\vec{\pi}})]N + \frac{g_A}{2f_\pi}N^\dagger\vec{\sigma} \cdot \vec{\nabla}(\vec{\tau} \cdot \vec{\pi})N, \quad (1)$$

where the  $\pi\pi N$  term is the Weinberg-Tomozawa (WT) interaction[16] and the  $\pi N$  term is the usual interaction that depends on the axial-vector coupling constant,  $g_A(\simeq 1.25)$ , and the pion-decay constant,  $f_\pi(\simeq 92 \text{ MeV})$ . Terms with additional pions or nucleons are neglected here, as they only contribute to the nuclear force at higher orders. The WT term has a specific normalization ( $-1/4f_\pi^2$ ) required by the underlying chiral symmetry.

The first treatments of the box (Fig. (1a)) and crossed-box (Fig. (1b)) diagrams that led to an energy-independent potential were performed by Taketani, Machida, and Ohnuma (TMO)[17], and by Sugawara and Okubo (S-O)[18]. Phenomenological Lagrangians of undetermined size were also incorporated by S-O, including a term of WT type. The first calculation based on a chiral Lagrangian (the new twist) was performed in Ref.[6], and this has been repeated by several groups[19, 20, 21]. The result (short-range terms have been ignored) is conveniently expressed in terms of isospin factors as

$$V_{2\pi} = V_{2\pi}^0 + V_{2\pi}^1 \vec{\tau}_1 \cdot \vec{\tau}_2, \quad (2a)$$

$$V_{2\pi}^0 = -\frac{m_\pi}{8\pi^3} \left( \frac{g_A m_\pi}{2f_\pi} \right)^4 \left( S_{12} \left[ \frac{12K_0(2x)}{x^3} + \frac{K_1(2x)}{x^2} \left( 4 + \frac{15}{x^2} \right) \right] - 4 \vec{\sigma}_1 \cdot \vec{\sigma}_2 \left[ \frac{3K_0(2x)}{x^3} + \frac{K_1(2x)}{x^2} \left( 2 + \frac{3}{x^2} \right) \right] \right), \quad (2b)$$

$$V_{2\pi}^1 = \frac{m_\pi}{4\pi^3} \left( \frac{g_A m_\pi}{2f_\pi} \right)^4 \left( -\frac{1}{2} \left[ \frac{K_0(2x)}{x} \left( 4 + \frac{23}{x^2} \right) + \frac{K_1(2x)}{x^2} \left( 12 + \frac{23}{x^2} \right) \right] + \frac{2}{g_A^2} \left[ \frac{K_1(2x)}{x^2} + \frac{5K_2(2x)}{2x^3} \right] + \frac{K_2(2x)}{2g_A^4 x^3} \right), \quad (2c)$$

where  $x = m_\pi |\vec{r}_1 - \vec{r}_2| = m_\pi r$ ,  $\vec{\sigma}_i$  and  $\boldsymbol{\tau}_i$  are the usual (Pauli) spin and isospin operators of nucleon “ $i$ ”, while  $S_{12}$  is the conventional tensor operator and the  $K_n$  are irregular Bessel functions. Terms proportional to  $g_A^4$  arise from the box and crossed-box diagrams (Figs. (1a) and (1b)) with the iterated OPEP appropriately subtracted,  $g_A^2$  terms from the triangle diagrams (Fig. (1c)), and the  $g_A$ -independent terms from the “football” diagram (Fig. (1d)) constructed from two WT interactions.

The chiral Lagrangian also contains interactions with more derivatives or powers of the quark masses, which include new seagull vertices giving rise to a subleading TPEP of the triangle type. Such forces have also been calculated[6, 20, 22], and some components (such as the central isoscalar) are known to be important[20, 23]. A new Nijmegen phase-shift analysis of  $pp$  data[23] has shown that this TPEP (leading plus subleading order) provides a better fit than the OPEP alone, and even better than the OPEP supplemented by heavier-meson exchange. In anticipation of a re-analysis of  $np$  data, we examine here CIB in the TPEP.

The pion-mass difference corresponds to a Lagrangian term that arises primarily from EM interactions,

$$\mathcal{L}_{\text{EM}}^{(1)} = -\frac{\delta m_\pi^2}{2} (\boldsymbol{\pi}^2 - \pi_3^2). \quad (3)$$

The superscript (1) here reflects the expected size of this term compared with the  $(\pi^0)$  mass in Eq. (1): on the basis of dimensional analysis[8, 9],  $\delta m_\pi^2/m_\pi^2 \sim (\alpha/\pi)(\Lambda/m_\pi)^2$ , which numerically is  $\sim (m_\pi/\Lambda)[5]$ . Combining the mass terms in Eqs. (1) and (3) produces different masses for the  $\pi^\pm$  and the  $\pi^0$  and correspondingly different pion propagators. Isospin violation arising from differing pion masses can then be implemented in a straightforward way by tagging pion masses in the propagators of Fig. (1) with the isospin labels at the vertices that created them. A straightforward (but tedious) calculation using the techniques of Ref.[19] leads to the results shown below. In the sense of power counting, these contributions are of the same order as the isospin-symmetric, subleading TPEP already incorporated in the new Nijmegen phase

shift analysis[23]. A much simpler derivation (than using tagged pion masses) is possible that is based on symmetries, and that is what is presented next. This derivation subsumes our leading-order result and applies to subleading orders, as well.

The defining aspect of the problem is the equality of  $\pi^+$  and  $\pi^-$  masses, which follows from CPT invariance, and this equality has been incorporated into Eq. (3). Under the reflection in the  $x$ - $z$  isospin plane that defines the charge-symmetry operation ( $x \rightarrow -x; z \rightarrow -z$ ), the interaction in Eq. (3) is invariant and therefore can generate only class (II) isospin violation, which is charge symmetric. For two nucleons (labeled 1 and 2) there is a unique isospin operator with this structure,

$$T_{20}(1, 2) = \tau_1^z \tau_2^z - \frac{\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{3}. \quad (4)$$

This is an isotensor with vanishing value for  $T = 0$  states, while for  $T = 1$  systems it has equal values for  $pp$  and  $nn$  channels (2/3) and a different value ( $-4/3$ ) for the  $np$  channel. We write a class (II) potential as

$$V_{\text{II}} = T_{20}(1, 2) \Delta V^{\text{CIB}}. \quad (5)$$

To illustrate our method, let us consider first the well-known effect of the pion-mass difference in the OPEP[14]. It follows from the fact that there are two charged and one neutral pion. If one expands the exact OPEP to first order in the pion-mass difference,  $\delta m_\pi$ , about an average pion mass

$$\overline{m}_\pi = \frac{2}{3}m_{\pi^+} + \frac{1}{3}m_{\pi^0}, \quad (6)$$

one easily finds an isospin-symmetric, isovector  $V_\pi^1(\overline{m}_\pi)$  plus the CIB piece

$$\Delta V_\pi^{\text{CIB}} = - \left( \frac{g_A}{2f_\pi} \right)^2 \frac{\delta m_\pi}{4\pi} \vec{\sigma}_1 \cdot \vec{\nabla} \vec{\sigma}_2 \cdot \vec{\nabla} e^{-\overline{m}_\pi r}. \quad (7)$$

Alternatively, Eq. (7) follows from the  $pp$  or  $nn$  results (where charge conservation requires that the exchanged pion be neutral) by expanding  $\pi^0$ -exchange about  $\overline{m}_\pi$ . Equation (6) is a very commonly used prescription in nuclear force models.

The TPEP isospin structure is simplest for the  $pp$  or  $nn$  cases, where charge conservation requires that the pair of exchanged pions be either both neutral or both charged (i.e.,  $\pi^+\pi^-$ ). The isoscalar potential  $V_{2\pi}^0$  (Eq. (2b)) must arise from a trace, or a sum over charge states (2 charged pions for each neutral one). Thus, the actual form of  $V_{2\pi}^0$  is  $[\frac{2}{3}V_{2\pi}^0(m_{\pi^+}; m_{\pi^-}) + \frac{1}{3}V_{2\pi}^0(m_{\pi^0}; m_{\pi^0})]$  (where the masses of both exchanged pions have been explicitly labeled using an obvious notation), since there are two

charged-pion pairs for each neutral pair. This exact result can also be expanded to first order in  $\delta m_\pi$  about  $\overline{m}_\pi$ . The expression  $V_{2\pi}^0(\overline{m}_\pi; \overline{m}_\pi)$  then approximately equals the expanded expression (or equivalently that the term proportional to  $\delta m_\pi$  cancels out). Thus, using  $\overline{m}_\pi$  in  $V_{2\pi}^0$  is correct through  $\mathcal{O}(\delta m_\pi)$ .

The isospin-dependent terms can be deduced by examining the isospin structure of the WT interaction for emitting two pions with isospins  $\alpha$  and  $\beta$ :  $\tau^\gamma \varepsilon^{\gamma\alpha\beta}$ . The  $pp$  (or  $nn$ ) interaction requires  $\gamma = 3$ , and therefore  $\alpha$  and  $\beta$  must be 1 and 2 (in either order), requiring that both exchanged pions must be charged. The box diagrams (and also diagrams involving any seagulls, including higher-order ones) have similar structures. In general, two pions emitted sequentially on a single nucleon line have an isospin factor  $\tau^\alpha \tau^\beta$  (for pions with isospin  $\alpha$  and  $\beta$ ) that can be decomposed into two irreducible terms:  $\delta^{\alpha\beta}$  and  $\varepsilon^{\alpha\beta\gamma} \tau^\gamma$ . When contracted with the corresponding factors on the second nucleon, the  $\delta^{\alpha\beta}$  leads to the isoscalar force discussed above and the second factor generates the isovector one proportional to  $\tau_1^z \tau_2^z$ . These are also the allowed structures for seagulls. Thus, our argument applies to any  $2\pi$ -exchange force of this type.

Summarizing, only a pair of charged pions can be exchanged in the isovector ( $\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$ ) part of the force appropriate for two protons or two neutrons. Incorporating the  $np$  case, the structure turns out to be

$$\begin{aligned} V_{2\pi}^1 &\sim \tau_1^z \tau_2^z V_{2\pi}^1(m_{\pi^+}; m_{\pi^+}) + (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 - \tau_1^z \tau_2^z) V_{2\pi}^1(m_{\pi^+}; m_{\pi^0}) \\ &\rightarrow 2 V_{2\pi}^1(m_{\pi^+}; m_{\pi^0}) - V_{2\pi}^1(m_{\pi^+}; m_{\pi^+}), \end{aligned} \quad (8)$$

where the second term on the first line contributes only to  $np$  interactions, and the second form applies only to the  $T = 1$   $np$  case. The integral corresponding to  $V_{2\pi}^1(m_{\pi^+}; m_{\pi^0})$  is intractable. However, if one expands the second form about  $\overline{m}_\pi$  one finds that to  $\mathcal{O}(\delta m_\pi)$  it is equivalent to  $V_{2\pi}^1(m_{\pi^0}; m_{\pi^0})$ , which is a standard integral. Thus, exchanging two neutral pions in the isovector part of the force is an excellent approximation for the  $T = 1$   $np$  case. For  $T = 0$  there is no CIB and Eq. (8) (top line) leads to  $V_{2\pi}^1(\overline{m}_\pi; \overline{m}_\pi)$  as an excellent approximation.

In the  $pp$  case ( $\tau_1^z \tau_2^z \rightarrow 1$  and  $T_{20}(1, 2) \rightarrow 2/3$ ) if the pion masses are expanded about  $\overline{m}_\pi$ , the isospin-violating force can be immediately deduced, since the part of  $V_{2\pi}^1$  proportional to  $\delta m_\pi$  is isospin violating

$$V_{2\pi}^1(m_{\pi^+}; m_{\pi^+}) \cong V_{2\pi}^1(\overline{m}_\pi; \overline{m}_\pi) + \frac{\delta m_\pi}{3} \frac{\partial}{\partial \overline{m}_\pi} V_{2\pi}^1(\overline{m}_\pi; \overline{m}_\pi), \quad (9)$$

where the first term is isospin conserving. Because of the  $2/3$  value of  $T_{20}$ , we arrive at the simple result

$$\Delta V_{2\pi}^{CIB} = \frac{\delta m_\pi}{2} \frac{\partial}{\partial \overline{m}_\pi} V_{2\pi}^1(\overline{m}_\pi; \overline{m}_\pi). \quad (10)$$

Incorporating the  $np$  interaction leads to the same result, which was verified by following the isospin of each exchanged pion in Fig. (1) in a conventional derivation[19].

Using the TMO potential in Eq. (2c) and performing the derivative in Eq. (10) one finds

$$\Delta V_{2\pi}^{CIB} = -\frac{\delta m_\pi}{4\pi^3} \left( \frac{g_A m_\pi}{2f_\pi} \right)^4 \left( -\frac{1}{2} \left[ \frac{4K_0(2x)}{x} + K_1(2x) \left( 4 + \frac{11}{x^2} \right) \right] + \frac{3K_1(2x) + 2x K_0(2x)}{g_A^2 x^2} + \frac{K_1(2x)}{2g_A^4 x^2} \right). \quad (11)$$

Terms proportional to  $g_A^4$ ,  $g_A^2$ , and  $g_A^0$  are again box, triangle, and football contributions. Equation (10), which leads to Eq. (11) in leading order of ChPT, is our primary result. To  $\mathcal{O}(\delta m_\pi)$  Eq. (11) is identical to that of Ref.[14] for the box diagrams, which were also calculated using the TMO approach. We note that our mass-expansion technique is similar to that of Ref.[24].

Although these forces are appropriate for the Nijmegen phase-shift analysis, which uses only the tail of the force, a complete potential must be regularized by a short-range cut off. We have employed the representations of the appendix of Ref.[19] in an attempt to generate analytic forms for the regularized potential, but have failed. Presumably either purely numerical forms of  $\Delta V_{2\pi}^{CIB}$  must be generated or approximations made to simplify intractable integrals.

We have resorted instead to an *ad hoc* cutoff procedure. We calculate the CIB effect using the charge-independent part of the AV18 potential[25] with the electromagnetic corrections turned off, and we use that potential's multiplicative two-pion-range cutoff:  $(1 - e^{-2.1r^2})^2$ . We find that the separate contributions of the [box,triangle,football] potentials to  $\Delta a = |a_{np}| - \frac{1}{2}|a_{pp} + a_{nn}|$  are: [0.98,-0.31,-0.02] fm for a total of 0.65 fm. The triangle contribution from the WT Lagrangian is a sizable correction to the dominant box graphs. We are aware of only two previous comparable calculations of the CIB from the pion-mass difference in two-pion-exchange forces, and neither can be directly compared to our result. Li and Machleidt[13] find 0.16 fm from the box graphs, but their calculation includes the (cutoff) delta functions that we have eliminated by renormalization. Ericson and Miller[26] find 0.88 fm using the relativistic PS-coupling model of Partovi and Lomon[27]. Although the latter is in reasonable agreement with our results, differences in the two approaches may make this agreement accidental.

In summary, for the first time the leading-order (static) chiral CIB  $NN$  force from  $2\pi$  exchange has been developed, employing both symmetry arguments and direct calculation of Feynman diagrams. The symmetry arguments apply only to the

CIB from the pion-mass difference, but are appropriate to any order in ChPT. We find that to  $\mathcal{O}(\delta m_\pi)$  the effective pion mass to be used in the isoscalar force or the  $T = 0$  force is  $\overline{m}_\pi$ , while  $m_{\pi^\pm}$  should be used for the  $pp$  or  $nn$  cases and  $m_{\pi^0}$  for the  $T = 1$   $np$  case.

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of the one-pion-exchange potential (see Ref.[19] for a complete historical discussion), which is an energy-dependent potential. Such a form leads to a different two-pion-exchange potential than that produced by the more usual energy-independent form of OPEP. In this paper we use the energy-independent form, as did Refs. [17, 18, 19, 20, 21, 22].

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